# APPLICATION OF MARKOV CHAIN MODELTO FOREIGN DEBT MANAGEMENT IN NIGERIAN ECONOMY

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**ABSTRACT**

Since the last century, there have been marked changes in the approach to scientific enquiries and greater realisation that probability (or non-deterministic) models are more realistic than deterministic models in many situations. Observations taken at different time points rather than those taken at a fixed period of time began to engage the attention of probabilists. Many stochastic processes occurring in social sciences are studied now not only as a random phenomenon but also as one changing with time or spacecalled Markov chain.This study considered an application of Markov chain model to predict future debt pattern as effective management of any nation's debt is crucial to growth and development of the economy of that nation. We collected data on debts maintained by all the thirty six states governments and Federal Capital Territory, Abuja in Nigeria for a period of six years and determined future debt trend based on the transition probabilities between various groups of transition states. Three noticeable transition states namely; rising, stable and dropping of debt trend were used. The findings revealedthat 55% of states governments will have a rise in their debt profile, 6% will have a stable debt profile while 39% will have a drop in their debt profile. Therefore, it was recommended that the federal government should design appropriate financing strategies that guarantee a debt path matchingloan with the ability to repay and put up a policy as a preventive action to reduce, in the medium term, the possibility of debt unsustainability by the various states governments in Nigeria.

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## INTRODUCTION

Statisticians and economists have for many hundreds of years assessed different methods for acquiring models that can give effective returns for better economy. Debt management is another area where statistical tools are used to model the debt profile pattern of a given nation. Foreign debt is one of the sources of financing capital formation in any economy. It is generally expected that Nigeria and other developing countries facing scarcity of capital will acquire external debt to supplement domestic saving (Pattilo *et al.*, 2002).

Adepoju *et al.*, (2007) noted that developing countries in Africa are characterized by inadequate internal capital formation due to vicious circle of low productivities, low incomes and low savings. Therefore, this situation calls for technical, managerial and financial support from western countries to bridge the resources gap.

The origin of Nigeria external debt dates back to 1958, when a sum of US $28million was used to finance the Nigerian Railways construction. Foreign borrowing was minimal between 1958 and 1977; and not the level that could be regarded as a burden to the country. Debt contracted during the period were the concessional loans from official sources such as World Bank and Nigeria's major trading partners with longer repayment periods from ten to forty years and lower interest rates.

External debt management refers to the establishment of the condition of issue and redemption of foreign loans. It involves a conscious and carefully planned schedule of the acquisition and retirement of loans contracted either for developmental purposes or to support the balance of payments. It makes use of estimates of foreign earnings, sources of exchange finance, project returns from the investments and repayment schedule. It also includes an assessment of the country's capacity to service existing debts and a judgement on the desirability of contracting loans (CBN, 1996).

The concept of debt itself involved dualistic character. That is, it involves internal debt and external debt. Whichever, debt taken by government has its own problems. Internal debt problem involves crowd out effect in the private sector due to higher interest rate as government borrowed domestically. If government borrows externally, it is faced with the problems of hard lending terms/conditions, higher foreign interest rates and short term rescheduling arrangement from its creditors.

The scope of applications of random variables which are functions of time or space or both has been on the increase and often times; mathematical models are used as main tools for making informed decisions. Markov chains are often useful in constructing a mathematical model of a situation involving experiments with multiple outcomes where the outcome of a given trial depends only on the outcome of the previous trial. When the outcome of a given experiment can affect the outcome of the next experiment, the process is referred to as a Markov chain, or Markov process (Ryan, 1973).

Mendehall *et al.* (1986) and Agbadudu (1996) consider a set of states and the ways in which the outcome move from one state to another. To begin the process of moving from one state to another, we begin in some initial state and continue the process by moving from one state to another. Each move from one state to another is called a step. Given some set of states $\{s_1, s_2, \dots, s_n\}$, if we are currently in some state $s_i$, the probability that our next step will be to another state $s_j$ is given by a probability denoted $p_{ij}$. These probabilities are called transition probabilities.

Since, in a Markov process, the outcome of the next experiment is not affected by any outcome other than the outcome of the current experiment, the probability of moving from some state $s_i$ to another state $s_j$ depends only on the current state $s_i$ and our probability distribution, not on any previous state of the process. Observe that the process can remain in the current state $s_i$ with probability $p_{ii}$. To start the Markov process, in addition to specifying a particular starting or initial state, we also need to specify an initial probability distribution. We can represent the probability distributions for a given initial state in a square matrix called a transition matrix (Gantmacher, 1959 and Ross, 1997).

Summarily, families of random variables which are functions of say, time are known as stochastic processes (or random processes, or random functions). A stochastic process $\{X_n, n = 0, 1, 2, \dots\}$ is called a Markov chain, if for $j, k, j_1, \dots, j_{n-1} \in N$, $P(X_n = k \mid X_{n-1} = j, X_{n-2} = j_1, \dots, X_0 = j_{n-1}) = P_{jk}$. Markov chain is a vital instrument for decision-making process, based on the past which can affect the present and the future. The outcomes are called the states of the Markov chain. So, if $X_n$ has the outcome j, the process is said to be at state j at nth trial. To a pair of states (j, k) at the two successive trials, there is an associated conditional probability $P_{jk}$ called transition probabilities and they are basic to the study of the structure of the Markov chain (Hogg and Craig, 1978; and Medhi, 2009).

Hence, earlier choices affect the present, while current decisions may influence the future, and so on. This cause and effect system on decision-making can be viewed as embedded in an unbounded horizon. Horizon may be viewed as a projection into the future of a given problem such as debt management of a given country (Abdulazeez, 2013). Therefore, the application of Markov chain in the analysis of foreign debt management in Nigeria is paramount and considered in this paper.

**LITERATURES REVIEW**

The use of Markov chain has received a new impetus and is at the front burners of debt analysis. Dryden (1969) and Ryan (1973) conducted investigations, in which aggregate and individual debt data of United Kingdom was analysed within a Markovian framework, and which indicated that it might be fruitful to apply the Markov model to more disaggregated data, specifically to debt data. Jarrow *et al.* (1997) presented a contingent claims valuation model that explicitly incorporates credit rating information into the valuation methodology. Altman *et al.* (1992) addressed the importance of the economic sector of the firm in the credit migration process. The null hypothesis that the credit transition matrix is constant across business sectors was confirmed using the same three business sectors of industrial, financial and utilities by Duffee (1998).

Markov chainis a special class of mathematical techniques applicable to decision problems. Such applications include manpower planning, assessing the behaviour of stock prices, estimating bad debts or credit management (Agbadudu, 1996). A Markov chain is a series of states of a system that has the Markov property. At each time the system may have changed from the state it was in the moment before, or it may have stayed in the same state. This change of state is called transitions. If a sequence of states has the Markov property, it means that every future state is conditionally independent of every prior state given the current state (Obodos, 2005).

## OBJECTIVES

The broad objective of this paper is to model dynamic of foreign debt in Nigerian economy using Markov chain. To complement this, the specific objectives include:

(i) Using Markov chain model to derive the probability distribution of next period debt given present time debt.
(ii) Considering the probability distribution of the debt management pattern.
(iii) Using Markov chain to successfully predict the future direction of debt pattern and to suggest ways to reduce debt management in Nigeria.

## DATA USED

The scope of this study includes all the thirty-six (36) states governments and the Federal Capital Territory (FCT), Abuja. We considered the debt maintained for the period of six years between 2007 and 2013as collected from the Debt Management Office, Abuja; and found inAbdulazeez (2013).

## METHODOLOGY

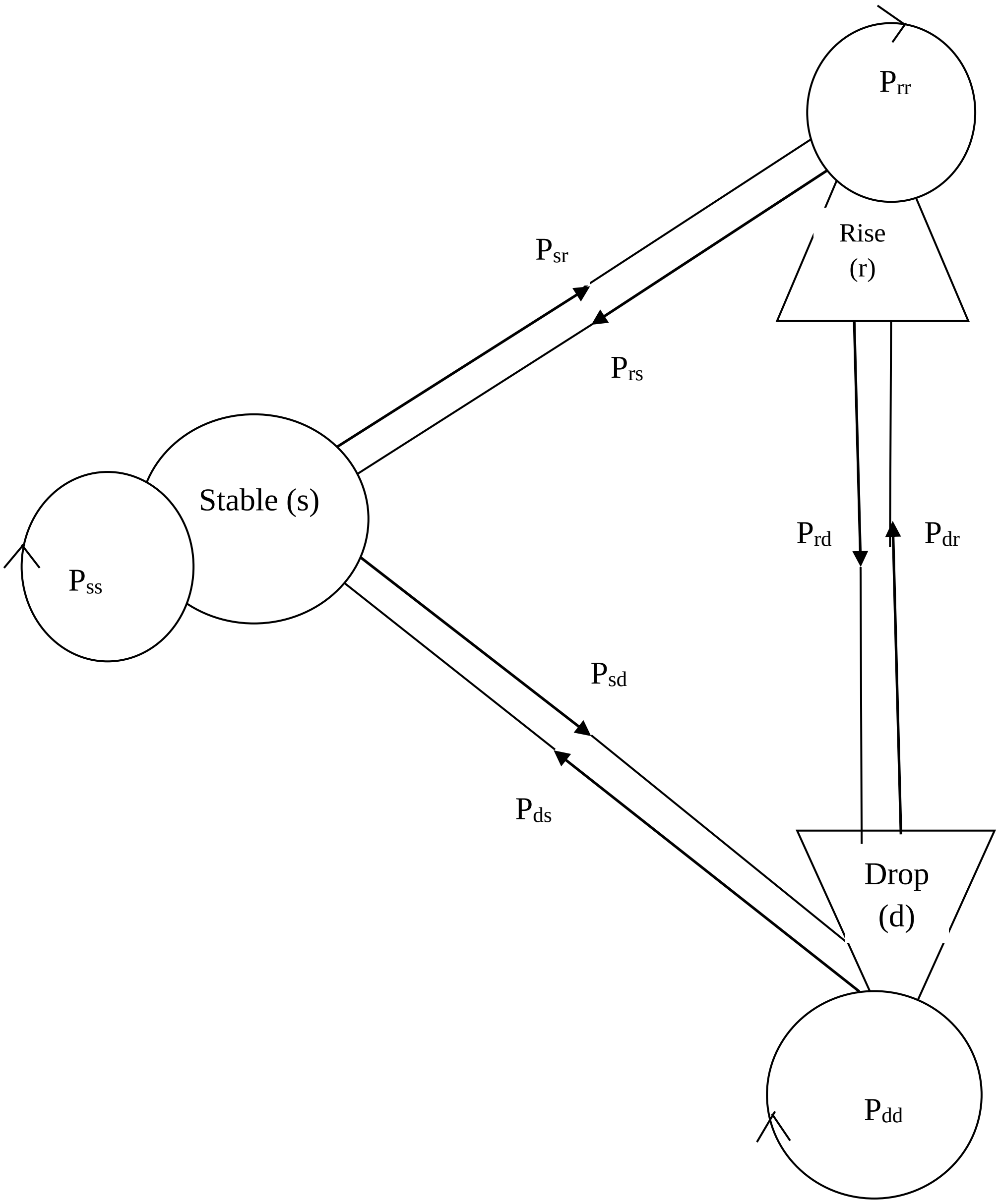
Assumptions of the model include:

i. Unit of time is one year
ii. In a given year, if the debt profile is in a rising state, the probability that a transition is made to a stable state or dropping state is dependent on the present state (rising) (Markov Model Assumption).
iii. The movement of Nigeria's Foreign Debt between the three states is considered as a random variable indexed by time parameter simply described as a stochastic process
iv. The following are the assumed states for the process
    a. Stable State: debt value does not change from current year
    b. Rising State: debt value changes upward from current year
    c. Drop State: debt value changes downward from current year

Thus we have, State 1: debt profile rising
State 2: debt profile stable
State 3: debt profile dropping

We considered foreign debt management in Nigeria as a three-state Markov chain model described as rising (r), dropping (d) and stable (s) as shown in Figure 1. With this, we derived the probabilities of the debt rising, dropping or remaining stable, and on the basis of these probabilities, predicted the future debt directions.

**Figure 1:** Transition Diagram for Debt Management




From Figure 1, we recorded the transition probabilities matrix *"P"* for the process as;

$$P = \begin{bmatrix} P_{rr} & P_{rd} & P_{rs} \\ P_{dr} & P_{dd} & P_{ds} \\ P_{sr} & P_{sd} & P_{ss} \end{bmatrix}$$

To find out whether the transition probabilities are stationary or not with time, we test for independence of $P_{ij}^{(k)}$ on k. Thus, we have;

$H_0 : P_{ij}^{(k)} = P_{ij}$ as against $H_1 : P_{ij}^{(k)}$ depend on k$\forall$ $i, j = 1,2,3$

That is, the null hypothesis is that the transition probability is independent on k

We used the maximum - likelihood ratio test for the estimation of transition probabilities from individual or micro-unit data.

$$M = \sum_{k=1}^{6} M_k = [f_{ij}]$$

where $f_{ij} = \sum_{k=1}^{6} f_{ij}(k)$

And the maximum - likelihood estimate of the stationary transition probability matrix is;

$$P_{ij}^{n} = \frac{f_{ij}}{f_i}$$

where $f_i = \sum_{j=1}^{3} f_{ij}$

The likelihood ratio criterion is given by;

$$\boldsymbol{\lambda} = \prod_{i,j=1}^{3} \prod_{k=1}^{6} [\frac{P_{ij}}{P_{ij}(k)}]^{f_{ij}(k)}$$ (1)

$$-2\log\lambda = \chi^2_{n(n-1)(t-1)}$$ (2)

where n is the number of states and t is the time parameter. We evaluate $\boldsymbol{\lambda}$ and calculate $-2\log\lambda$ which has an asymptotic $\chi^2$-distribution with n(n-1)(t-1) degree of freedom. With the acceptance of the $H_0$, we have homogeneous Markov chain model. The model is represented by a single transition count matrix and the $P_{ij}^{\prime s}$ are estimated. The n-step transition probabilities is determined by $P^n = P^0P^n$ (3)

Otherwise, we have non-homogeneous Markov chain model and the n-step transition probabilities is determined by
$P^n = P^0P^n$ with $P^0 = P_1P_2P_{3\ldots}P_6$ (4)

## RESULTS

### Transition Count

The following transition counts were recorded for the thirty-six (36) states governments of Nigeria and FCT over a six – year period spanning 2007 to 2013 (Tables 1 – 6).Markov chain requires the process to change at a given unit of time interval. One unit of time is one year as mentioned earlier under the assumption.

**Table 1: Transition Count for 2007 – 2008**

| | State 1 (r) | State2 (s) | State3 (d) | Total |
|---|---|---|---|---|
| State 1 (r) | 17 | 9 | 11 | 37 |
| State 2 (s) | 7 | 10 | 20 | 37 |
| State 3 (d) | 19 | 3 | 15 | 37 |
| | 43 | 22 | 46 | 111 |

**Table 2: Transition Count for 2008 – 2009**

| | State 1 (r) | State 2 (s) | State 3 (d) | Total |
|---|---|---|---|---|
| State 1 (r) | 3 | 20 | 14 | 37 |
| State 2 (s) | 14 | 9 | 14 | 37 |
| State 3 (d) | 3 | 14 | 20 | 37 |
| | 20 | 43 | 48 | 111 |

**Table 3: Transition Count for 2009 – 2010**

| | State 1 (r) | State 2 (s) | State 3 (d) | Total |
|---|---|---|---|---|
| State 1 (r) | 26 | 3 | 8 | 37 |
| State 2 (s) | 9 | 7 | 21 | 37 |
| State 3 (d) | 8 | 10 | 19 | 37 |
| | 43 | 20 | 48 | 111 |

**Table 4: Transition Count for 2010 – 2011**

| | State 1 (r) | State 2 (s) | State 3 (d) | Total |
|---|---|---|---|---|
| State 1 (r) | 9 | 9 | 19 | 37 |
| State 2 (s) | 10 | 0 | 27 | 37 |
| State 3 (d) | 8 | 1 | 28 | 37 |
| | 27 | 10 | 74 | 111 |

**Table 5: Transition Count for 2011 – 2012**

| | State 1 (r) | State 2 (s) | State 3 (d) | Total |
|---|---|---|---|---|
| State 1 (r) | 26 | 2 | 9 | 37 |
| State 2 (s) | 12 | 8 | 17 | 37 |
| State 3 (d) | 7 | 28 | 2 | 37 |
| | 45 | 38 | 28 | 111 |

**Table 6: Transition Count for 2012 – 2013**

| | State 1 (r) | State 2 (s) | State 3 (d) | Total |
|---|---|---|---|---|
| State 1 (r) | 27 | 2 | 8 | 37 |
| State 2 (s) | 17 | 0 | 20 | 37 |
| State 3 (d) | 23 | 4 | 10 | 37 |
| | 67 | 6 | 38 | 111 |

**Statistical Inference for Stationarity of the Transition Probability Matrix**

The estimated transition count matrix is obtained as;

$$M = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = \begin{pmatrix} 108 & 45 & 69 \\ 70 & 33 & 119 \\ 68 & 60 & 94 \end{pmatrix}$$

The estimated transition probabilities matrix is given by;

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{pmatrix} 0.486 & 0.203 & 0.311 \\ 0.315 & 0.149 & 0.536 \\ 0.306 & 0.270 & 0.423 \end{pmatrix}$$

Calculating $P^n$, we have

$$P^2 = \begin{pmatrix} 0.395 & 0.213 & 0.392 \\ 0.364 & 0.231 & 0.405 \\ 0.363 & 0.217 & 0.419 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.379 & 0.218 & 0.403 \\ 0.314 & 0.218 & 0.408 \\ 0.373 & 0.219 & 0.406 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.376 & 0.218 & 0.405 \\ 0.375 & 0.219 & 0.406 \\ 0.374 & 0.218 & 0.405 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.406 \\ 0.374 & 0.218 & 0.404 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \\ 0.374 & 0.217 & 0.404 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \\ 0.374 & 0.217 & 0.404 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \\ 0.375 & 0.218 & 0.405 \end{pmatrix}$$

$$= \begin{pmatrix} 0.38 & 0.22 & 0.41 \\ 0.38 & 0.22 & 0.41 \\ 0.38 & 0.22 & 0.41 \end{pmatrix} \text{ approximate to 2dp}$$

Thus, as n > 1, $P^n \rightarrow (0.38 \quad 0.22 \quad 0.41)$

This reveals that over the years, 38% of the states governments in Nigeria will have their debt profile rising, 22% will be stable while 44% will be dropping.

We wish to test for the null hypothesis that the chain is stationary;

Ho: $P_{ij}^k = P_j,\ i, j = 1, 2.$

We have;

$$\boldsymbol{\lambda} = \prod_{i,j=1}^{3} \prod_{k=1}^{6} \left[P_{ij} \middle| P_{ij}^k\right]^{f_{ij}^{(k)}}$$

$$= 6.078976\ x\ 10^{-16}$$

Then

$$-2 \ln \lambda = -2 \ln(6.078976\ x\ 10^{-16})$$

=70.07

P($\chi_{30}^2$≥70.07)being very small, the null hypothesis that the chain is stationary is rejected. The process is not stationary and its distribution is functionally dependent on time t.

Hence, we shall consider the non – stationary Markov chain.The maximum - likelihood estimate of the transition probabilities matrix$P_1P_2P_3P_4P_5P_6$ is;

$$P = P_1P_2P_3P_4P_5P_6$$

$$= \begin{pmatrix} 0.460 & 0.243 & 0.297 \\ 0.189 & 0.270 & 0.541 \\ 0.514 & 0.081 & 0.405 \end{pmatrix} \begin{pmatrix} 0.081 & 0.541 & 0.378 \\ 0.378 & 0.244 & 0.378 \\ 0.081 & 0.378 & 0.541 \end{pmatrix} \begin{pmatrix} 0.703 & 0.081 & 0.216 \\ 0.243 & 0.189 & 0.567 \\ 0.216 & 0.270 & 0.514 \end{pmatrix}$$

$$\begin{pmatrix} 0.243 & 0.243 & 0.514 \\ 0.270 & 0.000 & 0.730 \\ 0.216 & 0.627 & 0.757 \end{pmatrix} \begin{pmatrix} 0.703 & 0.054 & 0.243 \\ 0.324 & 0.216 & 0.460 \\ 0.189 & 0.757 & 0.054 \end{pmatrix} \begin{pmatrix} 0.730 & 0.054 & 0.216 \\ 0.460 & 0.000 & 0.540 \\ 0.622 & 0.108 & 0.270 \end{pmatrix}$$

$$= \begin{pmatrix} 0.568 & 0.032 & 0.399 \\ 0.568 & 0.32 & 0.399 \\ 0.567 & 0.31 & 0.4 \end{pmatrix}$$

Thus,

$$P^2 = \begin{pmatrix} 0.567 & 0.059 & 0.399 \\ 0.567 & 0.059 & 0.399 \\ 0.566 & 0.059 & 0.399 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.559 & 0.059 & 0.393 \\ 0.559 & 0.059 & 0.393 \\ 0.559 & 0.059 & 0.393 \end{pmatrix}$$
$$P^4 = \begin{pmatrix} 0.551 & 0.058 & 0.388 \\ 0.551 & 0.058 & 0.388 \\ 0.551 & 0.058 & 0.388 \end{pmatrix}$$
$$P^5 = \begin{pmatrix} 0.553 & 0.058 & 0.386 \\ 0.553 & 0.058 & 0.386 \\ 0.553 & 0.058 & 0.386 \end{pmatrix}$$
$$P^6 = \begin{pmatrix} 0.55 & 0.06 & 0.39 \\ 0.55 & 0.06 & 0.39 \\ 0.55 & 0.06 & 0.39 \end{pmatrix} \text{ approx to 2dp}$$

$$\therefore \prod_0 = \prod_0 P = (0.55 \quad 0.06 \quad 0.39)$$
$$\prod_1 = \prod_0 P_1 = (0.55 \quad 0.06 \quad 0.39)$$
$$\prod_2 = \prod_1 P_2 = (0.55 \quad 0.06 \quad 0.39)$$
$$\prod_3 = \prod_2 P_3 = (0.55 \quad 0.06 \quad 0.39)$$
$$\prod_4 = \prod_3 P_4 = (0.55 \quad 0.06 \quad 0.39)$$
$$\prod_5 = \prod_4 P_5 = (0.55 \quad 0.06 \quad 0.39)$$
$$\prod_6 = \prod_5 P_6 = (0.55 \quad 0.06 \quad 0.39)$$

Thus, 55% of the states governments in Nigeria will have a rise in their debt profile, 6% will have stable debt profile while 39% will have a drop in their debt profile.

**DISCUSSION**

The current situation as collected from National Debt Management Office, Abuja shows that 23 states corresponding to about 62% of the total number of states including the FCT have a rising debt profile while 12 states corresponding to 32% havea dropping debt profile and 2 states corresponding to 6% are maintaining a stable debt profile. Comparing these findings with the results in this paper, it is sufficient to say that our resultsare effective and could be used to predict future debt management pattern in Nigeria and beyond.

**CONCLUSION**

The result shows that Nigeria is a debtor country with more than fifty percent of the nation's states governments having debt rise now and in the future. The indicators provided in this study constitute a key element of broad policy design that involves the determination of the fiscal stance of the government and appropriate financing terms to reduce the possibility of debt unsustainability. The huge indebtedness has direct effect on the provision and maintenance of basic infrastructures in the country and the related poor living condition.

**RECOMMENDATIONS**

The pattern of debt profile in this study could be used as a tool for discussions with creditors on the volume and terms of financing to aid in identifying financial assistance needs in the area of debt management. The government should optimally provide additional capital as a means of production instead of issuing debt and hold assets instead of debt. There should be maintenance of prudent fiscal discipline and implementation of debt management strategy that places emphasis on prudent and productive public sector borrowing.

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